An effective compromise between these two objectives can be achieved by using ratio control on both the recycle and underflow streams. No sludge storage is required, variations in the sludge height are slight, and fluctuations in the reactor solids concentration are small relative to the uncontrolled case.

ACKNOWLEDGMENT

This work was supported by the U.S. Department of the Interior under the Water Resources Research Act of 1964, Public Law 88-379, as amended. We have had many useful discussions with our colleagues C. A. Petty and H. W. Blanch.

NOTATION

C = concentration
G = solids flux
h = height of a layer in settler
H = height of sludge blanket
k = constant in reaction rate
Q = volumetric flow rate
r = volumetric recycle ratio from settler
R = reaction rate
t = time
u = underflow velocity from settler
v = solids settling velocity in settler
V = reactor volume
w = sludge volumetric wasting rate
Y = yield factor
δ = velocity of solids discontinuity in settler
μ = substrate dependent portion of reaction rate
$\hat{\mu}$ = coefficient in reaction rate
τ = time constant for reaction rate

Subscripts

f = feed
n = layer number in settler
r = recycle
s = substrate (BOD)
x = solids

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Countercurrent Backmixing Model for Slugging Fluidized-Bed Reactors

A model is developed for slugging fluidized bed catalytic reaction which improves on the model of Hovmand and Davidson by incorporating the solids mixing. The model parallels the countercurrent backmixing model of Fryer and Potter. Solids mixing is accounted for by the model of Thiel and Potter which assumes the rising slug is followed by a well-mixed wake into and out of which solids flow at a rate determined by the rise velocity of the slug. Comparison is made with experimental data reported by Hovmand and Davidson with good results.

JANAKIRAMAN RAGHURAMAN

and

O. E. POTTER

Department of Chemical Engineering
Monash University
Clayton, 3168 Victoria, Australia

SCOPE

The design of commercial size, gas fluidized-bed reactors is generally based on data obtained from reactors of the laboratory or the pilot plant scale. In these smaller reactors, slug flow often occurs owing to rapid coalescence of the bubbles above the distributor plate, particularly when the height of the bed is large compared to its diameter. Therefore, it is necessary to understand the effects of slug flow on conversion, so that laboratory or pilot plant data may be used for scaling-up.

Hovmand and Davidson (1971) proposed a two-phase model for slugging beds, similar to the two-phase model for bubbling beds by Davidson and Harrison (1963). Com-

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parison of the model with the data on ozone decomposition showed significant deviations at high flow rates and larger values of the reaction rate constants.

Fryer and Potter (1972, 1974, 1976) have shown that the countercurrent backmixing model provides a good description of the behavior of bubbling fluidized beds, with advantages over other models. A three-phase model based on the countercurrent backmixing model has been developed here for describing the slugging fluidized-bed reactor. The model gives better prediction of the data than the two-phase model of Hovmand and Davidson and is based on the solids mixing model of Thiele and Potter (1977).

CONCLUSIONS AND SIGNIFICANCE

It seems axiomatic that models of fluidized-bed reactors should take into account the mixing processes in the solids and the effect of these mixing processes on the gas. A suitable model which meets this criterion is proposed and offers significant advantages.

In roasting and fluidized combustion processes, what happens to the fresh feed assumes some importance, and it is therefore desirable that a reactor model should include a description of the solid mixing. Since the present model is derived from a description of solid mixing, it could be used for noncatalytic heterogeneous reactions with the appropriate modifications.

Furthermore, an advantage of the new model is that it provides a basis for a mechanistic study of dynamic stability, and although such a study will have direct application only to slugging fluidized-bed reactors, it will give some important indications as to the dynamic behavior of beds in which the bubble diameter is very much less than the vessel diameter.

Slugging fluidized beds are of importance not only because some industrial beds, particularly those of large aspect ratio, operate in the slugging state, but also because most pilot plant reactors operate in this state, and scale-up demands a good understanding of the behavior of slugging-bed reactors, particularly as compared with bubbling reactors where the bubble diameter is much less than the diameter of the vessel.

The only explicit study of the slugging fluidized-bed reactor has been made by Hovmand and Davidson (1971) who report experimental data and compare the data with a two-phase model, modified by an assumption of complete mixing at the point or points in the bed where coalescence occurs. The two-phase model assumes piston flow of slug gas at $(U - U_{mf})$ and piston flow through the particulate phase at U_{mf}, mass exchange between gas in slugs and gas in the particulate phase being predicted by a model which takes account of diffusion and convection. In the present work, the Hovmand-Davidson analysis of gas exchange is retained. What is new in the present work is the incorporation of a model to describe the solid mixing as developed by Thiel and Potter (1977). These authors suppose the rising slug is followed by a wake of solids which is perfectly mixed, and this model gives an adequate representation of the solids mixing. It is then assumed that the gas in this wake region is also well mixed. The resultant model shows many similarities to the countercurrent backmixing model, Fryer and Potter (1976). The flow of solids in and out of the wake region is more easily calculated in the slugging model, since it is simply related to the rise velocity of the slug.

The slugging state is a limiting case of the bubbling bed, where the bubble is large enough to bridge the diameter of the vessel.

Hovmand and Davidson (1971) proposed a two-phase model for a slugging bed. The model assumes that there

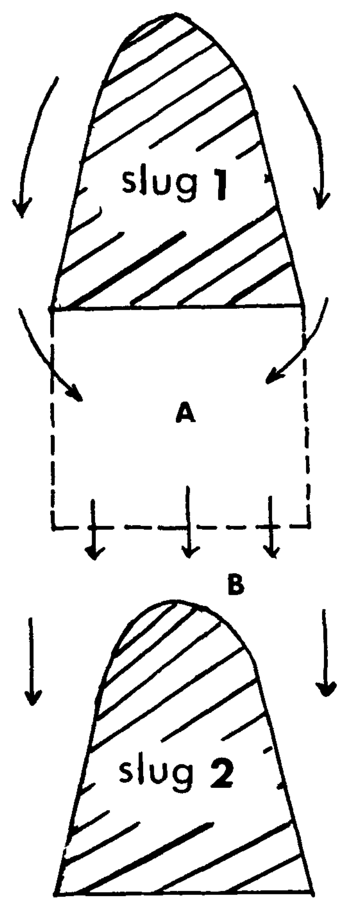

Fig. 1. Schematic representation of the slugging fluidized bed model. A: wake region in which solids and associated gas are perfectly mixed. B: region in which solids and associated gas are in plug flow.

exist two phases in the reactor, the slug phase and the particulate phase. Gas ecxhange occurs between the phases owing to diffusion and bulk flow. The authors compared the model predictions with the experimental data of Hovmand and Davidson (1968) and Hovmand, Freedman, and Davidson (1971). At higher gas velocities and for larger values of the reaction velocity constant, the deviation was considerable. Furthermore, it was assumed that complete mixing occurs when the slugs coalesce, and there is little or no evidence for this assumption.

PROPOSED MODEL

Thiel and Potter (1977) showed that a good representation of solids mixing data in slugging fluidized beds is obtained by making the assumption that the solids flowing around the slug flow into a well-mixed wake region and thereafter into a plug flow region, so that the bed is made up of slugs followed by a well-mixed wake, followed by a plug flow region. With reference to Figure 1, solids and associated gas flow past slug 1 and mix into the circulating wake region A. The wake is assumed to have a shape similar to that observed behind a solid object in a liquid and to extend back to the nose of the following slug (slug 2). Solids and associated gas flow out of the wake region A into region B, where virtually no mixing occurs, region B extending down past slug 2. It is this well-mixed wake followed by a piston flow region that is considered in the present work.

The model bears close resemblance to the countercurrent backmixing model of Fryer and Potter (1972, 1976) for bubbling beds. The slugging bed consists of the slugs, a wake of solids, carried upward by the slugs, and a particulate phase of solids flowing downward which corresponds to the piston flow section of the interslug region. The countercurrent backmixing model for the present situation is schematically portrayed in Figure 2.

GAS FLOWS

The cloud volume is lumped with the wake volume. The superficial fluidizing gas velocity is

$$U = U_{GB} + U_{GP} + U_{GC} \tag{1}$$

where subscripts B, P, and C refer to the bubble or slug phase, particulate phase, and cloud wake, respectively. The voidages in the wake and the particulate phases are assumed to be the same as those at incipient fluidization, namely, ϵ_{mf}.

The gas flow carried by the slug phase is given by

$$U_{GB} = U - U_{mf} \tag{2}$$

Since the absolute velocities of the slug and the cloud wake are the same

$$U_{GC} = f_w \, \epsilon_{mf} \, U_{GB} \tag{3}$$

From Equations (1) to (3) we get

$$U_{GP} = U_{mf} - \epsilon_{mf} f_w \, (U - U_{mf}) \tag{4}$$

The onset of backmixing will occur when the particulate gas flow reverses its direction of flow, or when $U_{GP} = 0$. Thus, the criterion of backmixing is given by

$$\frac{U_{cr}}{U_{mf}} = 1 + \frac{1}{f_w \epsilon_{mf}} \tag{5}$$

For $U > U_{cr}$, U_{GP} will be negative. This is the same criterion as advanced by Stephens, Sinclair, and Potter (1967) for the bubbling bed.

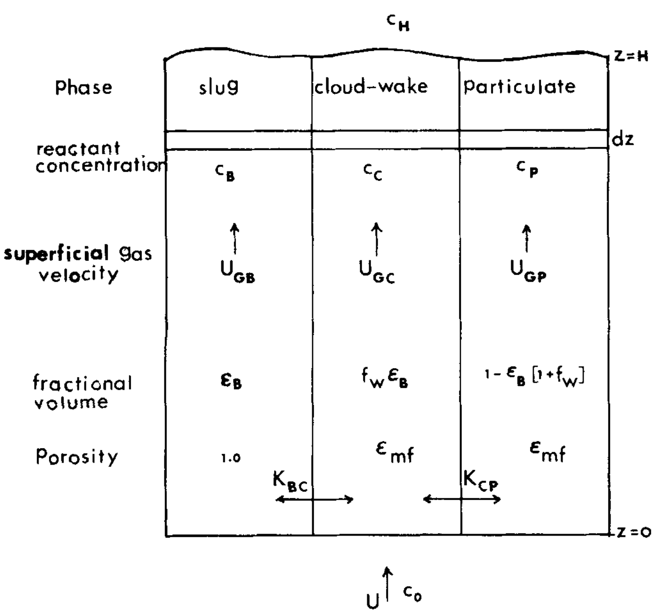

Fig. 2. Schematic diagram of the fluidized-bed reactor.

GAS EXCHANGE COEFFICIENTS

Hovmand and Davidson (1971) postulated that gas exchange between the slug and the particulate phase occurs owing to diffusion and bulk flow. Two cases were considered.

1. The gas exchange due to diffusion and bulk flow were additive. This resulted in the expression

$$\frac{Q}{V_s} = \frac{1}{Dm}\left[U_{mf} + \frac{16\epsilon_{mf} I}{1+\epsilon_{mf}}\left(\frac{D_G}{\pi}\right)^{1/2}\left(\frac{g}{D}\right)^{1/4}\right] \tag{6}$$

2. The diffusive and bulk flow components interacted with each other. The expression obtained by the authors was

$$\frac{Q}{V_s} = \frac{F_b}{Dm(F_b + \epsilon_{mf} F_p)}\left[U_{mf} + 16\epsilon_{mf} I \, F_p \left(\frac{D_G}{\pi}\right)^{1/2}\left(\frac{g}{D}\right)^{1/4}\right] \tag{7}$$

In Equations (6) and (7), m is a shape factor for the slug, given by

$$m = \frac{4V_s}{\pi D^3} = \frac{L_s}{D} - 0.495\left(\frac{L_s}{D}\right)^{1/2} + 0.061 \tag{8}$$

F_b and F_p are reduction factors given by Hovmand and Davidson (1971):

$$F_b = \exp\left(\frac{-B^2}{4D_G}\right) \Big/ \operatorname{erfc}\left(\frac{B}{2\sqrt{D_G}}\right) \tag{9}$$

$$B = 2U_{mf}\left(\frac{D}{3.832U_s}\right)^{1/2} \tag{10}$$

$$F_p = \exp\left(\frac{-P^2}{4D_G}\right) \Big/ \left[1 + \operatorname{erf}\left(\frac{P}{2\sqrt{D_G}}\right)\right] \tag{11}$$

and

$$P = B/\epsilon_{mf} \tag{12}$$

In Equations (6) and (7), I is a surface integral whose value is a function of the slug length to column diameter ratio as given in Table 1.

U_s is the rise velocity of an isolated slug given by (Kehoe and Davidson, 1970)

$$U_s = 0.35\,(gD)^{1/2} \tag{13}$$

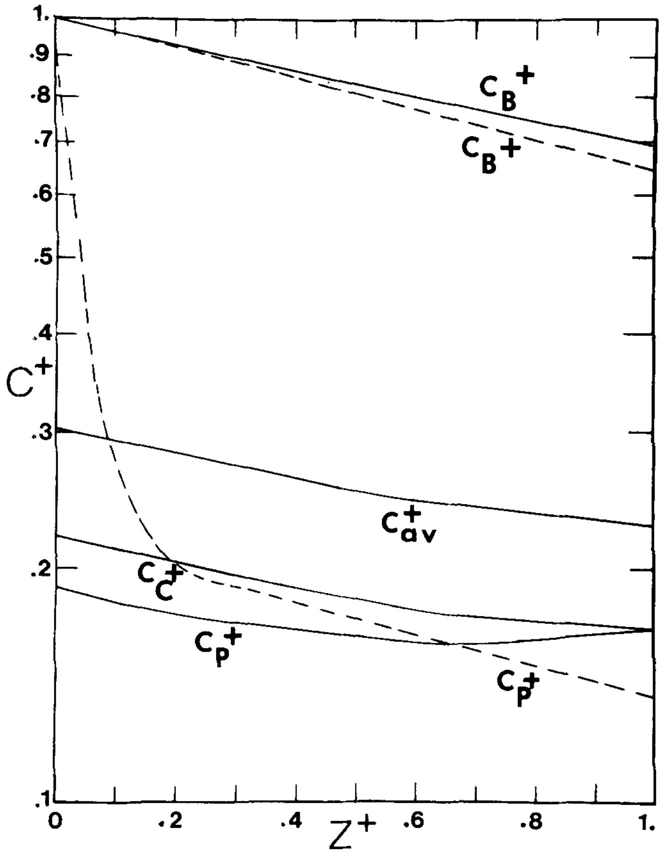

Fig. 3. Concentration profiles within the reactor. ——— present model; – – – – – Hovmand-Davidson model; $U_{mf} = 1$ cm s^{-1}; $U/U_{mf} = 10$; $D = 40$ cm; $H_{mf} = 200$ cm; $\epsilon_{mf} = 0.5$; $D_G = 0.2$ cm^2 s^{-1}; $T = 2$.

TABLE 1. VALUES OF SURFACE INTEGRAL I AS A FUNCTION OF L_s/D

L_s/D	0.3	0.5	1.0	2.0	3.0	4.0	5.0
I	0.13	0.21	0.39	0.71	0.98	1.24	1.48

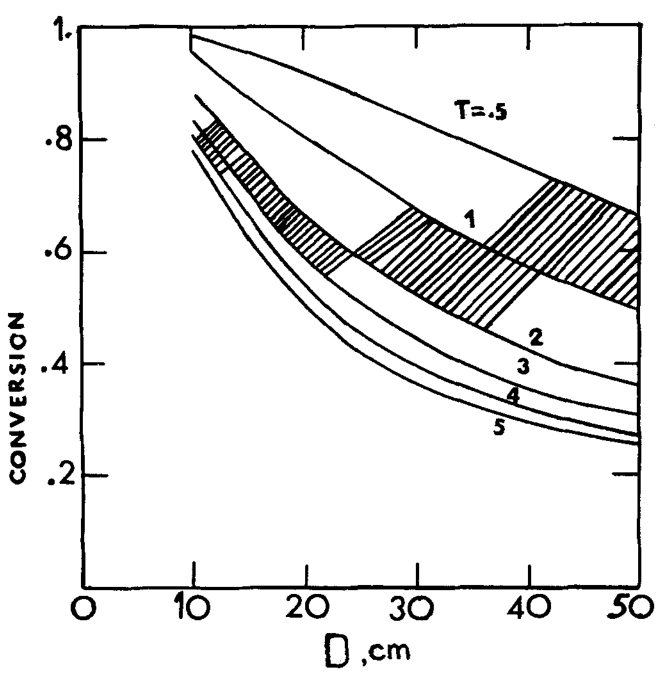

Fig. 4. Effect of interslug spacing on the conversion. $U_{mf} = 1$ cm s^{-1}; $U/U_{mf} = 10$; $H_{mf} = 200$ cm; $\epsilon_{mf} = 0.5$; $D_G = 0.2$ cm^2 s^{-1}.

The rise velocity of continuously generated slugs is given by

$$u_{sA} = U - U_{mf} + 0.35\,(gD)^{1/2} \tag{14}$$

The exchange coefficients Q/V_s given by Equations (6) and (7) are applicable for gas exchange between the slug and the cloud wake and will be denoted by K_{BC}. The exchange coefficient between the cloud wake and the particulate phase is obtained as follows.

Before we consider gas exchange in and out of the wake, we must first consider exchange of solids (and associated interstitial gas). Since the rise velocity of a slug is U_s, viewing the bed with the slug fixed and the solids moving downwards, the velocity of solids moving downwards is also U_s (Stewart and Davidson, 1967). The solids phase flows in and out of the wake, which is of volume $f_w V_s$, at a volumetric rate of $U_s \pi D^2/4$. This exchange can be expressed in terms of an exchange coefficient, namely, the volume of solids phase exchanged with the wake per unit slug volume per unit time, or

$$U_s\left(\frac{\pi D^2/4}{V_s}\right)$$

If there were no flow of gas through the solids phase, then the gas exchange rate would be

$$\epsilon_{mf}\left(\frac{U_s \pi D^2/4}{V_s}\right)$$

However, following Stewart and Davidson (1967), the flow out of the slug nose and the flow in at the base will be equal to $U_{mf}(\pi D^2)/(4)$. Therefore, the net volume swept by a rising slug, or the net volume of gas exchanged between the particulate phase and the wake per unit time, will be $(U_s\,\epsilon_{mf} - U_{mf})\,\pi D^2/4$. Hence

$$K_{CP} = \frac{(U_s\,\epsilon_{mf} - U_{mf})\,\dfrac{\pi D^2}{4}}{V_s} \tag{15}$$

If the fraction of the bed volume occupied by the slugs is ϵ_B, then

$$\epsilon_B = \frac{V_s}{V_s + \dfrac{\pi D^2}{4}\,(TD)} \tag{16}$$

Combining Equations (15) and (16), we get

$$K_{CP} = \frac{(1 - \epsilon_B)}{(TD)\,\epsilon_B}\,[U_s\,\epsilon_{mf} - U_{mf}] \tag{17}$$

CHEMICAL REACTION

The reaction is first order in gas concentration and occurs in the presence of solids in the particulate and cloud wake phases. The contact of reactant gas with the very small amount of solids which may be present in the slug is neglected. This assumption is reasonable for the reactions normally encountered (Kunii and Levenspiel, 1969). Also, the cloud volume is assumed negligible in comparison with the wake volume which is true for small U_{mf}.

The material balances on reactant gas over a differential element dz along the length of the reactor and in the slug, cloud wake, and particulate phases, respectively, are

$$U_{GB}\,\frac{dC_B}{dz} = K_{BC}\,(C_C - C_B)\,\epsilon_B \tag{18}$$

$$U_{GC}\,\frac{dC_C}{dz} = K_{CP}\,(C_p - C_C)\,\epsilon_B$$

$$+ K_{BC}(C_B - C_C)\epsilon_B - kf_w \epsilon_B C_C \quad (19)$$

$$U_{GP}\frac{dC_p}{dz} = K_{CP}(C_C - C_p)\epsilon_B - k[1 - \epsilon_B(1 + f_w)]C_p \quad (20)$$

We define the following dimensionless variables and groups:

$$C_B^+ = C_B/C_o; \quad C_C^+ = C_C/C_o; \quad C_p^+ = C_p/C_o;$$

$$z^+ = z/H; \quad X_{BC} = K_{BC}\,H_{mf}/U_s;$$

$$X_{CP} = K_{CP}\,H_{mf}/U_s; \quad U_{GB}^+ = U_{GB}/U; \quad U_{GC}^+ = U_{GC}/U; \quad U_{GP}^+ = U_{GP}/U \quad \text{and} \quad k' = k\,H_{mf}/U$$

Equations (18) to (20) become

$$U_{GB}^+\frac{dC_B^+}{dz^+} = \frac{X_{BC}\,U_s^+\,\epsilon_B}{(1-\epsilon_B)}(C_C^+ - C_B^+) \quad (21)$$

$$U_{GC}^+\frac{dC_C^+}{dz^+} = X_{CP}\frac{U_s^+\,\epsilon_B}{(1-\epsilon_B)}\left[(C_p^+ - C_C^+) + X_{BC}\frac{U_s^+\,\epsilon_B}{(1-\epsilon_B)}(C_B^+ - C_C^+)\right] - \frac{k'\,\epsilon_B}{(1-\epsilon_B)}f_w\,C_C^+ \quad (22)$$

$$U_{GP}^+\frac{dC_p^+}{dz^+} = X_{CP}\frac{U_s^+\,\epsilon_B}{(1-\epsilon_B)}(C_C^+ - C_p^+) - k'\frac{[1-\epsilon_B(1+f_w)]}{(1-\epsilon_B)}C_p^+ \quad (23)$$

These can again be written in a simplified form as

$$\frac{dC_B^+}{dz^+} = A_1\,C_B^+ + A_2\,C_C^+ \quad (24)$$

$$\frac{dC_C^+}{dz^+} = A_3\,C_B^+ + A_4\,C_C^+ + A_5\,C_p^+ \quad (25)$$

$$\frac{dC_p^+}{dz^+} = A_6\,C_C^+ + A_7\,C_p^+ \quad (26)$$

BOUNDARY CONDITIONS

The wake fraction f_w can be represented as a fraction of the interslug volume-slug volume ratio. Thus

$$f_w = \alpha\frac{\frac{\pi D^2}{4}TD}{V_s} \quad (27)$$

The only experimental evidence available for the value of α is due to Thiel (1972) and Thiel and Potter (1977), who give $\alpha = 0.7 \pm 0.2$. If we use this value, we find that for a value of $2U_{mf} < U < 3U_{mf}$, backmixing begins for most situations. Since most reactors operate at superficial gas velocity of greater than $2U_{mf}$, it is enough to consider the boundary conditions where backmixing occurs.

Following Fryer and Potter (1972), we have

$$\text{At} \quad z = 0, \quad C_B = C_o \quad (28)$$

$$-U_{GP}C_p + (U - U_{GB})C_o = U_{GC}C_C \quad (29)$$

and at

$$z = H, \quad C_C = C_p \quad (30)$$

The required expression for the exit gas concentration is

$$UC_H = U_{GB}\,C_B + (U - U_{GB})C_C \quad (31)$$

In dimensionless terms, Equations (28) to (31) become

$$\text{at} \quad z^+ = 0, \quad C_B^+ = 1 \quad (32)$$

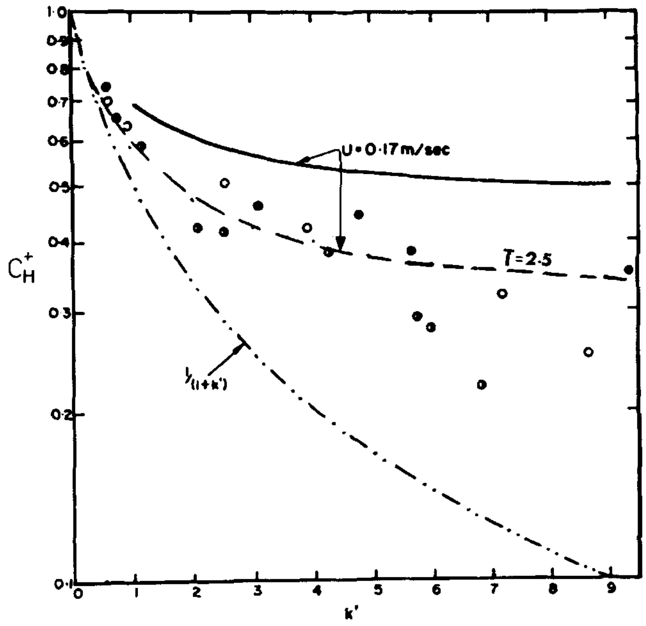

Fig. 5. Comparison of the model with the experimental data and model of Hovmand and Davidson. — — — present model; ——— Hovmand-Davidson, Equation (6); — · · — · · — CSTR; $U_{mf} = 1$ cm s^{-1}; $H_{mf} = 130$ cm; $\epsilon_{mf} = 0.5$; $D = 10$ cm.

$$-U_{GP}^+\,C_p^+ + (1 - U_{GB}^+) = U_{GC}^+\,C_C^+ \quad (33)$$

or

$$C_C^+ = B_1 + B_2\,C_p^+ \quad (34)$$

$$\text{At} \quad z^+ = 1, \quad C_C^+ = C_p^+ \quad (35)$$

and

$$C_H^+ = U_{GB}^+\,C_B^+ + (1 - U_{GB}^+)C_C^+ \quad (36)$$

THEORETICAL SOLUTION

The system of differential Equations (24) to (26), with the boundary conditions (32), (34), and (35), can be solved analytically to give

$$C_B^+ = \sum_{i=1}^{3} R_i \exp(\lambda_i z^+) \quad (37)$$

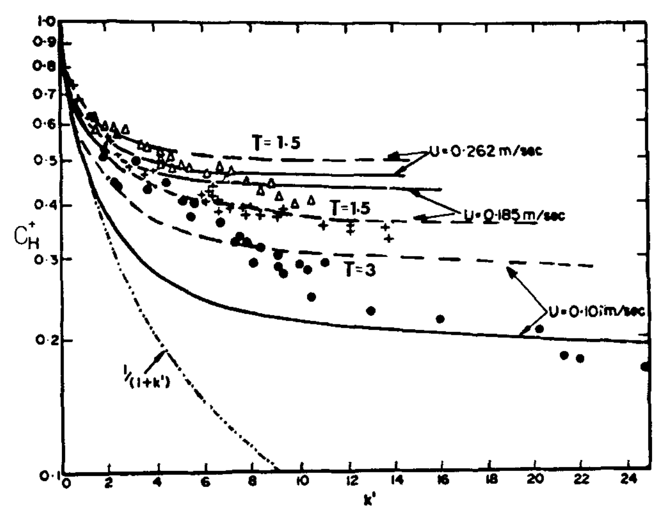

Fig. 6. Comparison of the model with the experimental data and model of Hovmand and Davidson. — — — present model; ——— Hovmand-Davidson, Equation (6); — · — · — CSTR: $U_{mf} = 3$ cm s^{-1}; $H_{mf} = 255$ cm; $\epsilon_{mf} = 0.48$; $D = 46$ cm; Δ $U = 0.262$ m s^{-1}; + $U = 0.185$ m s^{-1}; ● $U = 0.101$ m s^{-1}.

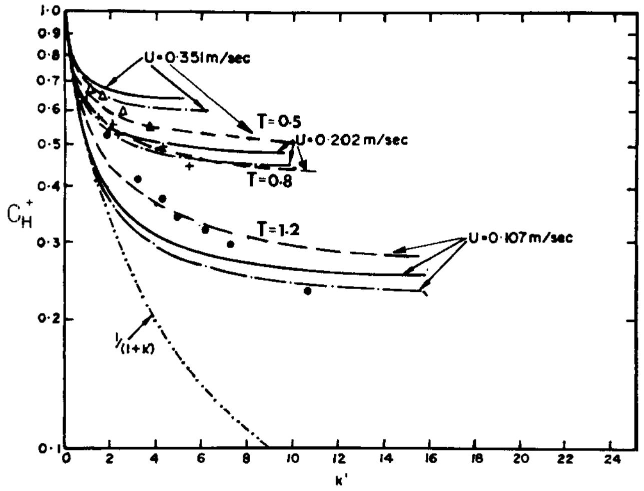

Fig. 7. Comparison of the model with the experimental data and model of Hovmand and Davidson. - - - present model; ——— Hovmand-Davidson, Equation (6); —·—·— Hovmand-Davidson, Equation (7); —·· —··— CSTR; $U_{mf} = 3$ cm s^{-1}; $H_{mf} = 129$ cm; $\epsilon_{mf} = 0.48$; $D = 46$ cm; Δ $U = 0.351$ m s^{-1}; $+$ $U = 0.202$ m s^{-1}; ● $U = 0.107$ m s^{-1}.

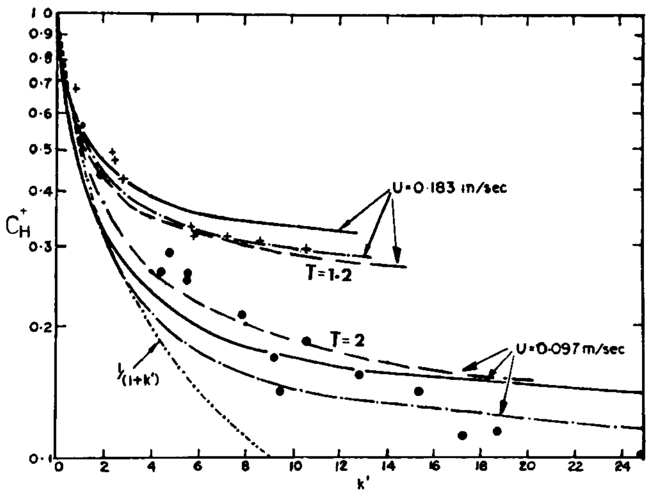

Fig. 8. Comparison of the model with the experimental data and model of Hovmand and Davidson. - - - present model; ——— Hovmand-Davidson; Equation (6); —·—·— Hovmand-Davidson, Equation (7); —·· —··— CSTR; $U_{mf} = 3$ cm s^{-1}; $H_{mf} = 288$ cm; $\epsilon_{mf} = 0.48$; $D = 46$ cm; $+$ $U = 0.183$ m s^{-1}; ● $U = 0.097$ m s^{-1}.

$$C_C{}^+ = \sum_{i=1}^{3} \alpha_i R_i \exp(\lambda_i z^+) \tag{38}$$

and

$$C_p{}^+ = \sum_{i=1}^{3} \beta_i R_i \exp(\lambda_i z^+) \tag{39}$$

The exit concentration may be obtained by evaluating Equations (37) and (38) at $z^+ = 1$ and substituting in Equation (36).

CONCENTRATION PROFILES

Figure 3 shows one set of concentration profiles predicted by the proposed model. The predictions of the Hovmand-Davidson model are also shown alongside for comparison. It is to be noted that in the absence of coalescence data, a value of $T = 2$ is used for both the proposed model and that of Hovmand and Davidson, in obtaining the concentration profiles in Figure 3. It is seen that the concentrations predicted by the simplified model are fairly close to those of the three-phase model. The particulate phase concentration of the two-phase model falls in between the wake and particulate phase concentrations of the three-phase model, in the central part of the reactor. Also, Equation (6) was used for evaluating the exchange coefficient.

The effect of column diameter on conversion is shown in Figure 4. A range value of T has been used for generating the family of curves. However, since the interslug spacing is dependent on factors like fluidizing velocity, the column diameter, particle size, and the height of the unfluidized bed, a profile in Figure 4 for any given value of T is not applicable for the entire range of column diameters. Furthermore, very little information is available on the variations of the interslug spacing with the system variables. Following the data of Kehoe and Davidson (1970) and Thiele (1972), who have given some values for the variation of T with D, the shaded area in Figure 4 represents the range within which the conversion would lie. Values of T for columns of 10 and 20 cm diameters were taken from Thiele (1972), and, for a column of 46 cm, they were obtained by curve fitting the conversion data of Hovmand, Freedman, and Davidson (1971).

Figures 5 to 9 show the comparison between the experimental data on ozone decomposition obtained by Hovmand and Davidson (1968) and Hovmand, Freedman, and Davidson (1971) with the predictions of the countercurrent backmixing model. The predictions of the Hovmand and Davidson (1971) model are also shown alongside for comparison. It is seen that the countercurrent backmixing model predictions are closer to the experimental data than those of the simple two-phase model. In the absence of decisive experimental evidence on the dependence of T on the system variables, the value of T was adjusted to obtain a close fit. It can be noted that the range of variation of T obtained by such fit corresponds with the range indicated by Kehoe and Davidson (1970) and Thiel (1972).

In obtaining the theoretical curves for the countercurrent backmixing model, Equation (6) was used for the exchange coefficient. However, Hovmand and Davidson (1971) have used Equation (6) in Figures 5 and 6 and

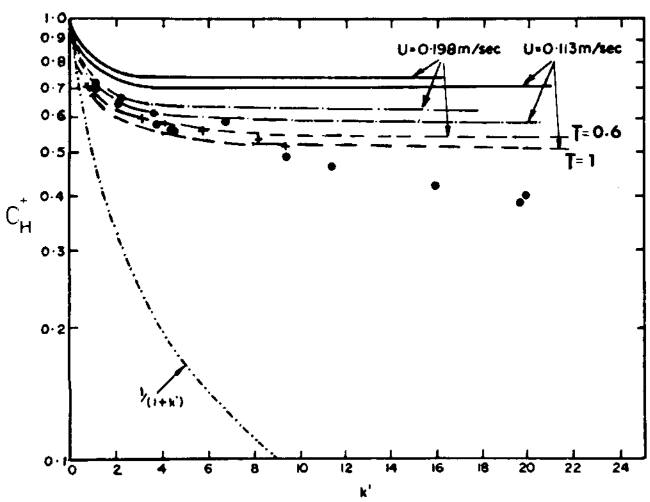

Fig. 9. Comparison of the model with the experimental data and model of Hovmand and Davidson. - - - present model; ——— Hovmand-Davidson, Equation (6); —·—·— Hovmand-Davidson, Equation (7); —·· —··— CSTR; $U_{mf} = 0.7$ cm s^{-1}; $H_{mf} = 260$ cm; $\epsilon_{mf} = 0.5$; $D = 46$ cm; $+$ $U = 0.198$ m s^{-1}; ● $U = 0.113$ m s^{-1}.

both Equations (6) and (7) in Figures 7 to 9. The authors, however, recommend the use of Equation (6). An inspection of Figures 7 to 9 reveals that the predictions of the two-phase model with Equation (7) are better than those with Equation (6), although both of them deviate from the experimental results. On the other hand, the use of Equation (6) throughout the predictions of the countercurrent backmixing model gives reasonable correspondence with the experimental findings. The deviation of the proposed model from the experimental data at large values of k' (fast reaction) may be due to the fact that the model assumes that slugs start forming right from the distributor, whereas in practice, there is a bubbling zone present above the distributor before slugging begins. A combination of the countercurrent backmixing model of Fryer and Potter (1972) for the bubbling part of the bed, followed by the proposed model for the slugging bed, would be expected to give better predictions of the experimental data for larger values of the dimensionless reaction velocity constant.

CONCLUSIONS

A three-phase model, similar to the countercurrent backmixing model for bubbling fluidized beds, has been proposed for slugging fluidized beds. The model is based on the solids mixing model for slugging beds, due to Thiele and Potter (1977). A comparison with the experimental data on ozone decomposition reveals that the proposed model is better capable of representing the experimental data than the simplified two-phase model of Hovmand and Davidson. A combination of the bubbling-slugging fluidized bed based on the concepts of backmixing appears to provide a promising description of the fluidized-bed reactor behavior.

NOTATION

$A_1 = -X_{BC}\,U_s^+\,\epsilon_B/U_{GB}^+(1-\epsilon_B)$

$A_2 = -A_1$

$A_3 = X_{BC}\,U_s^+\,\epsilon_B/U_{GC}^+(1-\epsilon_B)$

$$A_4 = -\frac{[X_{Cp}\,U_s^+\,\epsilon_B + X_{BC}\,U_s^+\,\epsilon_B + k'\,\epsilon_B f_w]}{U_{GC}^+(1-\epsilon_B)}$$

$A_5 = X_{CP}\,U_s^+\,\epsilon_B/U_{GC}^+(1-\epsilon_B)$

$A_6 = X_{CP}\,U_s^+\,\epsilon_B/U_{GP}^+(1-\epsilon_B)$

$$A_7 = -\frac{[X_{CP}\,U_s^+\,\epsilon_B + k'\{1-\epsilon_B(1+f_w)\}]}{U_{GP}^+(1-\epsilon_B)}$$

C_{avg} = average reactant concentration in bed, mole cm^{-3}
C_B = reactant concentration in bubble gas, mole cm^{-3}
C_C = reactant concentration in cloud wake gas, mole cm^{-3}
C_H = reactant concentration in exit gas, mole cm^{-3}
C_o = reactant concentration at inlet, mole cm^{-3}
C_p = reactant concentration in particulate phase gas, mole cm^{-3}
D = column diameter, cm
D_G = gas phase diffusivity, cm^2 s^{-1}
F_b = reduction factor defined in Equation (9)
F_p = reduction factor defined in Equation (11)
f_w = ratio of wake volume to slug volume
g = gravitational acceleration, cm s^{-2}
H = height of the expanded bed, cm
H_{mf} = height of incipiently fluidized bed, cm
I = integral given in Table 1
k = first-order reaction rate constant, based on unit volume of dense phase, s^{-1}
K_{BC} = volumetric rate of gas exchange between bubble (slug) and cloud wake per unit bubble volume, s^{-1}
K_{CP} = volumetric rate of gas exchange between cloud wake and particulate phase per unit bubble volume, s^{-1}
k' = dimensionless reaction rate constant $k\,H_{mf}/U$
L_s = length of slug, cm
m = shape factor for slug volume, Equation (8)
Q = volumetric rate of gas exchange between bubble and particulate phase, cm^3 s^{-1}
T = interslug spacing to column diameter ratio
U = superficial velocity of fluidizing gas, cm s^{-1}
U_{cr} = superficial gas velocity above which backmixing occurs, cm s^{-1}
U_{GB} = superficial gas velocity in bubble phase, cm s^{-1}
U_{GC} = superficial gas velocity in cloud wake phase, cm s^{-1}
U_{GP} = superficial gas velocity in particulate phase, cm s^{-1}
U_{mf} = superficial gas velocity at incipient fluidization, cm s^{-1}
U_s = rise velocity of an isolated slug, cm s^{-1}
U_{SA} = rise velocity of continuously generated slugs, cm s^{-1}
V_s = slug volume, cm^3
X_{BC} = dimensionless exchange coefficient, $K_{BC}\,H_{mf}/U_s$
X_{CP} = dimensionless exchange coefficient, $K_{CP}\,H_{mf}/U_s$
z = height above the distributor, cm
α = fraction of interslug volume that forms the wake
ϵ_B = fraction of bed volume occupied by slugs
ϵ_{mf} = voidage of bed at incipient fluidization

Superscript

$+$ = dimensionless quantity

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